Bulletin of the Manifold Atlas - definition (2013)

Gluck construction - definition*

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1. Definition

Any orientation preserving self diffeomorphism of $S^1 \times S^2$ is either isotopic to identity or to the map $\varphi : S^2 \times S^1 \to S^2 \times S^1$ defined by $\varphi(x, y) = (\alpha(y)x, y)$, where $\alpha \in \pi_1(SO(3)) \cong \mathbb{Z}/2\mathbb{Z}$ is the generator (e.g. [6] p.232). For any smooth 4-manifold X, and an imbedded 2-sphere in $S \subset X$ with a trivial normal bundle, the operation of removing the regular neighborhood $\nu(S) \cong S^2 \times D^2$ of S from X and then regluing it via the nontrivial diffeomorphism:

$$X \mapsto X_S = (X - \nu(S)) \smile_\varphi (S^2 \times D^2)$$

is called the *Gluck twisting of X along S*. This operation was introduced in [5].

2. Examples

When X is described as a handlebody, and S is represented by a 2-handle attached along an unknotted circle with zero framing, then the handlebody of X_S is obtained from the handlebody of X by putting one full right (or left) twist to all of the attaching framed circles of the other 2-handles going through this circle.

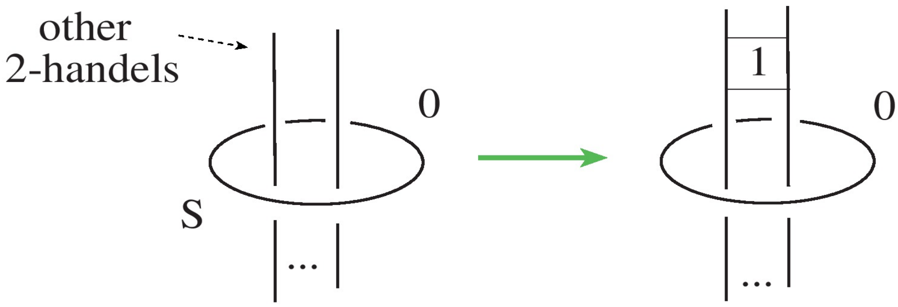

Figure 1. Figure 1

3. Some Results

It is known that $X_S \sharp P$ is diffeomorphic to $X \sharp P$, when P is a copy of $\mathbb{C}\mathrm{P}^2$ with either orientation. When S is null-homologous and X is simply connected this operation does not change the homeomorphism type of X. It is not known whether a Gluck twisting operation can change the diffeomorphism type of any smooth orientable manifold, while it is known that this is possible in the nonorientable case ([2]). In many instances Gluck twisting of manifolds appear naturally, where this operation do not change their diffeomorphism types (e.g. [5], [3], [4], [1]).

*Atlas page :www.map.mpim-bonn.mpg.de/Gluck_construction

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